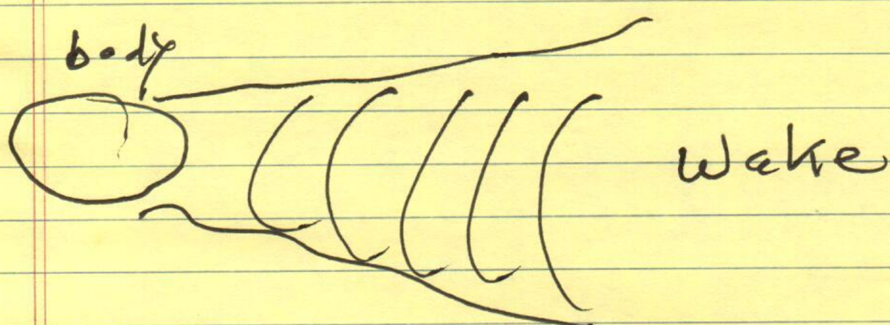
B.) Wakes - Simple physics   cf: Prandtl - Tietjens, Falkovich, Lander

Wake is:

- region of departure from potential flow behind object moving thru water and experiencing drag



- wake is ~~in~~ inexorably coupled to drag

- Message of wakes:

A little ~~r~~ forces a global adjustment of flow structure

- drag - thinking in frame where object at rest, drag results from loss of flow momentum to object.

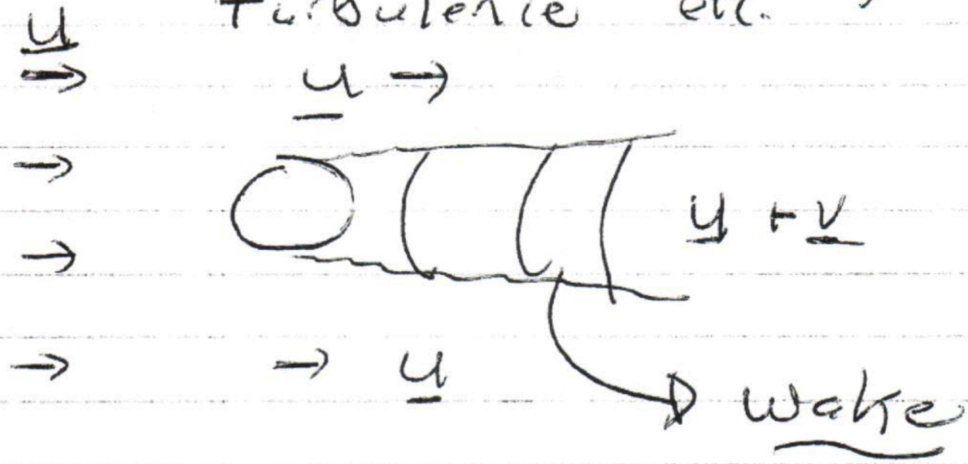
→ wake is region of flow where loss of momentum is evident.

i.e.,

– if potential flow (no drag)

symmetry upstream downstream in ⊥ displacement of fluid element

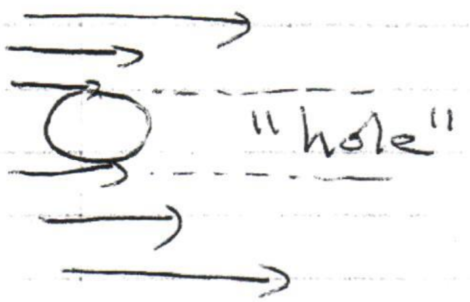
– with no-slip b.c., viscosity turbulence etc.



$\underline{v}$ opposite $\underline{u}$ i.e. [redacted]

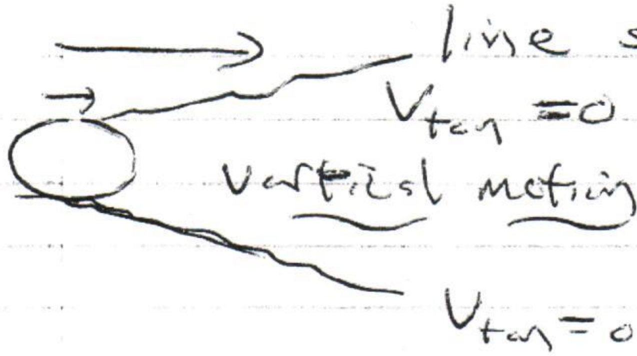
$\begin{cases} u>0 \\ v<0 \end{cases}$

- origin of wake is no-slip b.c. + $\left\{ \begin{array}{l} \text{viscosity} \\ \text{turbulence} \end{array} \right.$, after separation



but flow is unstable!

3D



$\underline{\omega} = \underline{\nabla} \times \underline{V} \neq 0$

- boundary of wake traced by fluid particles:

  → passing close to body

  → scattered by diffusion and turbulent mixing

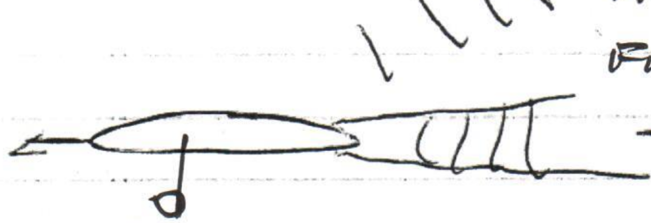
  ⇒ expansion

Note:

- in general, wake multi-component

Kelvin → waves

Finnegan's wake.



→ center-line

due: screw bubbles b.c. (skin friction)

S.S. Finnegan

Kelvin

- here, consider spherical cow of wake problems

→ ρ, ν

→ R

→ sphere

→

→

u

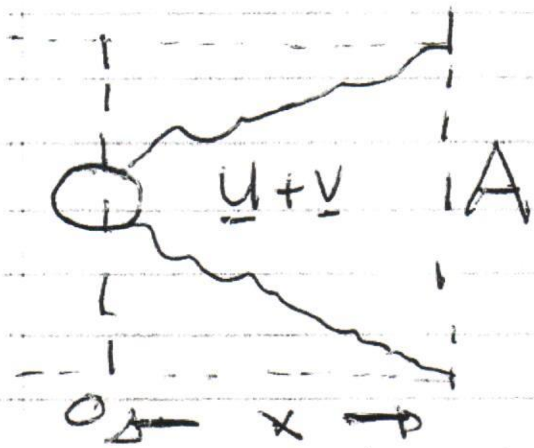
so $F_d \sim \rho u^2 R^2 f(Re)$

→ no surface effects.

→ How calculate wake structure?

Force of Drag ≡ Net Rate of Momentum Loss from Flow!

i.e.



downstream distance

Rate Momentum Loss ≡

$$A\,P_{total}(x) - A\,P_{Tot}(0) = F_d$$

$$P_{Tot}(0) \cong P + \rho U^2/2$$

total head.

$$P_{tot}(x) \cong P + \rho (U+V)^2/2$$

$$A \sim \pi\, w(x)^2$$

$w \equiv$ width of wake at x downstream

So $$F_d \approx w(x)^2 \left[\left(P + \frac{\rho}{2}(U+V)^2\right) - \left(P + \frac{\rho U^2}{2}\right) \right]$$

P unchanged → const straight streamlines

So

$$F_d \sim w(x)^2 \left[\cancel{p} + \cancel{\frac{\rho U^2}{2}} + \frac{2\rho U V_x}{2} - \cancel{p} - \cancel{\frac{\rho U^2}{2}} \right]$$

$$\sim \rho U V_x w(x)^2$$

n.b. why $p(0) \sim p(x)$?

$$\therefore \quad \boxed{F_d \sim \rho U V_x w(x)^2}$$

Now, need $w(x)$ to get V_x!

→ Observe:

- problem now reduced to one of scale
- wakes are self-similar!

$\Rightarrow$ $w \sim x^\alpha$, α ?

- wakes can be laminar [redacted] or turbulent

[redacted]

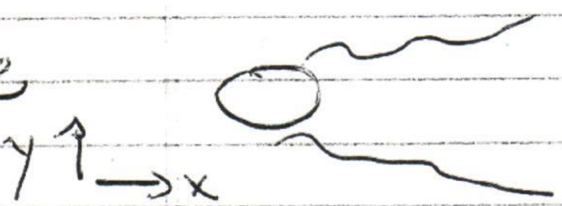
(i) Laminar $\qquad UR/\nu \ll 1$

now $\quad \cancel{\frac{\partial \underline{V}}{\partial t}} + \underline{V}\cdot\nabla\underline{V} - \nu\nabla^2\underline{V} = -\frac{\nabla P}{\rho}$

st. state

$$\underline{U}\cdot\nabla\underline{V} + \cancel{\underline{V}\cdot\nabla\underline{V}} - \nu\nabla^2\underline{V} = -\frac{\nabla P}{\rho}$$

so



$\nabla\cdot\underline{V} = 0$

$$U\partial_x V_y - \nu(\cancel{\partial_x^2} + \partial_y^2)V_y = -\frac{\partial_y P}{\rho}$$

and

$$U\partial_x V_x - \nu\partial_y^2 V_x = -\frac{\partial_x P}{\rho}$$

take $\quad \partial_x \sim 1/X$

$\qquad \partial_y \sim 1/W$

$$\left(\frac{U}{x} - \frac{\nu}{w^2}\right) v_y \sim -\frac{p}{w\rho}$$

$$\left(\frac{U}{x} - \frac{\nu}{w^2}\right) v_x \sim -\frac{p}{x\rho}$$

$$\underline{\nabla} \cdot \underline{v} = 0 \quad \Rightarrow \quad \frac{v_x}{x} \sim \frac{v_y}{w}$$

as p negligible (will show) $\Rightarrow$

$$\frac{U}{x} \sim \frac{\nu}{w^2}$$

$$\Rightarrow \boxed{w \sim (\nu x/U)^{1/2}}$$

$\rightarrow$ diffusive spreading of momentum by ν

$\rightarrow \sim (\nu t)^{1/2}$ with $t \sim x/U$.

so

$$W \sim \left(\frac{x}{R}\right)^{1/2}\left(\frac{\nu R}{U}\right)^{1/2}$$

$$\boxed{W/R \sim \left(\frac{x}{R}\right)^{1/2} 1/Re^{1/2}} \qquad \boxed{V_x \sim \frac{F_d}{\rho U W^2}}$$

→ akin Blasius B.L. thickness.

→ in case you are wondering:

y: $\frac{p}{\rho W} \sim \frac{\nu V_y}{W^2}$ (if assume)

and

$$\frac{V_x}{x} \sim \frac{V_y}{W}$$

$$\Rightarrow \quad p \sim \rho \nu V_x/x$$

and $p/\rho x \sim \frac{\nu V_x}{x^2} \ll \frac{\nu V_x}{W^2}$

drop p. and safely $\nu V_y/W^2$

(ii) Turbulent $\qquad Re \sim UR/\nu \gg 1$

$$\underline{U}\cdot\nabla\underline{v} + \underline{v}\cdot\nabla\underline{v} - \cancel{\nu\nabla^2\underline{v}} = -\frac{\nabla p}{\rho}$$

ignore

$$\Rightarrow \quad \frac{U}{X} v_x \sim \frac{\tilde{v}_y}{W} v_x$$

wave spreads by advection, not diffusion

$\tilde{v}_y \sim$ turbulent velocity $\qquad \boxed{W \sim \frac{\tilde{v}_y}{U} x}$

Take wake turbulence isotropic,

so $\tilde{v}_x \sim \tilde{v}_y$. $\qquad$ Fair? Test?

$$W \sim x\tilde{v}_x / U$$

but from drag:

$$\tilde{v}_x \sim F_d / \rho U W^2$$

$\Rightarrow$

$$w \sim x \frac{F_d}{\rho u^2 w^2} \sim x \left(F_d / \rho u^2 w^2 \right)$$

$$w^3 \sim F_d \, x / \rho u^2$$

$$\Rightarrow \boxed{\begin{aligned} w &\sim \left(F_d / \rho u^2 \right)^{1/3} x^{1/3} \\ &\sim \left(C_D R^2 \right)^{1/3} x^{1/3} \end{aligned}}$$

then, comparing widths:

$$\boxed{\begin{aligned} &\text{laminar:} \quad w/R \sim (x/R)^{1/2} \, Re^{-1/2} \\ &\qquad\qquad Re \sim UR/\nu \\ &\text{turbulent:} \quad w/R \sim (x/R)^{1/3} \, C_D^{1/3} \end{aligned}}$$

interestingly Laminar wake expands with downstream length more rapidly!

Why?

⇒ turbulence can relax ∇V behind object (due separation) rapidly, and faster than ν. Thus surrounding flow penetrates the dead water region more rapidly.

Also observe: <u>Wake</u> Re drops with x.

⇒

$$Re \sim \frac{W V_y}{\nu} \sim \frac{W V_x}{\nu} \sim \frac{\cancel{W}}{\nu} \frac{F_d}{\rho U W^{\cancel{2}}}$$

y direction (spr)

wake flow Re

$$Re \sim F_d / \rho U W \nu$$

$$\sim \frac{U^{\cancel{2}} R^2 \cancel{\rho_0} C_D}{\nu \cancel{\rho_0} \cancel{U} (C_D R^2)^{1/3} x^{1/3}}$$

$C_D \sim 1$

$$\sim \left(\frac{UR}{\nu}\right)\left(R/x\right)^{1/3}$$

$$\boxed{Re(x) \sim Re_0 \, (R/x)^{1/3}}$$

and $Re(x) \to 1$ at

$$x_L \sim R \, (Re_0)^3$$

distance behind boat where turbulent wake transitions to laminar.

i.e. akin l_d: transition from turbulent mixing to viscous mixing

N.B. In wake, vortical/rotational region can expand into irrotational region, but never reverse!

i.e. would really violate H-Thm ...

Wakes – Supplement   Skip

→ Revisit turbulent wake using turbulent viscosity, i.e.

$$W \sim \left(\nu x / U \right)^{1/2} \qquad (\nu \to D_T)$$

$$\to \left(D_T x / U \right)^{1/2}$$

i.e. is width of turbulent wake set by turbulent diffn, following Blasius Laws

but $D_T \sim W \tilde{V}$ ⇒ turbulent viscosity at mixing length level.

$$\sim W \left(F_d / \rho U W^2 \right)$$

$$\sim F_d / \rho U W \sim \text{const} / W$$

⇒

$$W \sim \left(F_d x / \rho U^2 W \right)^{1/2}$$

$$W^{3/2} \sim \left(F_d / \rho U^2 \right)^{1/2} x^{1/2} \sim \left(C_D R^2 \right)^{1/2} x^{1/2}$$

$$\sim C_D^{1/2} R \, x^{1/2}$$

$$W \sim \left(C_D \right)^{1/3} R^{2/3} x^{1/3}$$

$$\Rightarrow \boxed{W/R \sim C_D^{1/3} (X/R)^{1/3}}$$ agrees ✓.

Now, $D_T \sim \tilde{v} W$

$$\sim \frac{(\tilde{v} W^2)}{W}$$

$$\sim \frac{\rho U \tilde{v} W^2}{\rho U W}$$

$$\sim \frac{Q}{W} \sim Q / R (X/R)^{1/3}$$

∴ – Point is that turbulent viscosity, mixing drops downstream, relative to constant viscous mixing.

– follows from $\tilde{v} W \sim \overbrace{Q}^{\to \text{const.}} / W$

– explains why turbulent wake spreads more slowly than laminar wake.

→ Some Observations re: Wake Flows

→ note,

$$F_x = -\rho U \int_{\text{Wake}} v_x \, dy \, dz$$

Now $Q = \rho \int_{[redacted]} V_x \, dy \, dz$

↓

mass flow due wake

⇒ deficit.

→ but if encircle body

$$\rho \int \underline{v} \cdot d\underline{a} = 0$$ i.e. continuity!

Now total $\underline{V}$ → {velocity field / departure from $\underline{U}$}

= vortical wake flow + potential flow.

so, must have $\underline{V}_{\substack{pot\\flow}}$ s/t

$$\int \underline{V} \cdot da = Q/\rho$$

then, for area at r:

$$v\,\pi r^2 \sim Q/\rho$$

$$\left.\begin{aligned} &\Rightarrow\ v \sim Q/r^2 \\ &\phi \sim Q/r \end{aligned}\right\}$$

global adjustment in potential flow due wake/viscosity (localized)

Message:

A little ν forces a global adjustment in flow structure.